**Chapter # 06 (Trigonometric Function)**

**6.4 Graphs of the Sine and Cosine Functions:**

**Objectives:** 1 Graph Functions of the Form Using Transformations

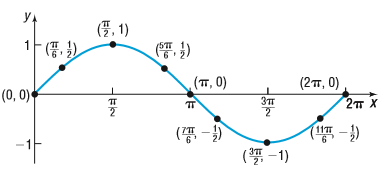
2 Graph Functions of the Form Using Transformations

3 Determine the Amplitude and Period of Sinusoidal Functions

4 Graph Sinusoidal Functions Using Key Points

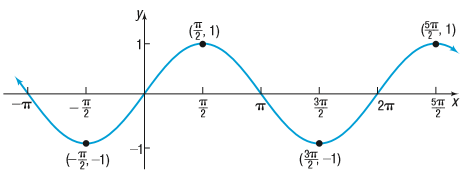
5 Find an Equation for a Sinusoidal Graph

**The Graph of the sine function, :** Because the sine function has period , it is only necessary to graph on the interval . The remainder of the graph will consist of repetitions of this portion of the graph.



**Figure:**

The graph in above figure is one period, or cycle, of the graph of . To obtain a more complete graph of , continue the graph in each direction, as shown in the following figure:



**Figure:**

**Properties of the sine function, :**

1. The domain is the set of all real numbers.

2. The range consists of all real numbers from -1 to 1, inclusive.

3. The sine function is an odd function, as the symmetry of the graph with respect to

the origin indicates.

4. The sine function is periodic, with period .

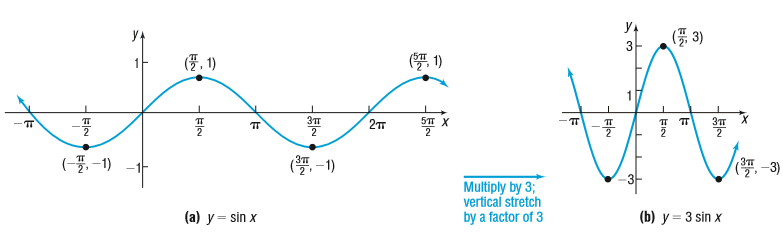
5. The *x*-intercepts are ; the *y*-intercept is 0.

6. The maximum is 1 and occurs at

the minimum value is -1 and occurs at

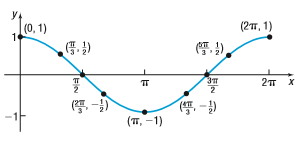
**Example 1:** Draw the graph of using transformations. Use the graph to determine the domain and the range of the function.

**Solution:**



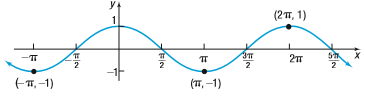
The domain of is the set of all real numbers, . The range is , or

**The Graph of the cosine function, :**



**Figure:**

A more complete graph of is obtained by continuing the graph in each direction, as shown in following figure.



**Figure:**

**Properties of the cosine function:**

1. The domain is the set of all real numbers.

2. The range consists of all real numbers from -1 to 1, inclusive.

3. The cosine function is an even function, as the symmetry of the graph with respect to

the *y*-axis indicates.

4. The cosine function is periodic, with period .

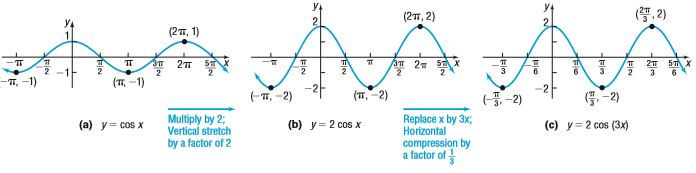
5. The x-intercepts are ; the *y*-intercept is 1.

6. The maximum value is 1 and occurs at

the minimum value is -1 and occurs at .

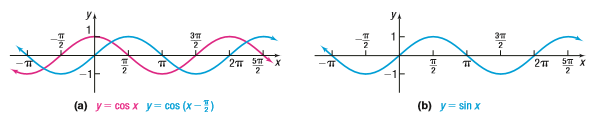
**Example 3:** Draw the graph of using transformations. Use the graph to determine the domain and the range of the function.

**Solution:**



The domain of is the set of all real numbers, . The range is , or . Notice in **figure (c)** that the period of the function is .

**Sinusoidal Graphs:** Shift the graph of to the right units to obtain the graph of , figure (a) . Now look at the graph of in figure (b). Notice th at the graph of is the same as the graph of .



Based on the above figure, we conjecture that

**Theorem:** If , the amplitude and period of and are given by: Amplitude = and Period = *T* =

**Example 4:** Determine the amplitude and period of .

**Solution:** Comparing to , note that *A* = 3 and .

Amplitude=, Period =

**Steps for Graphing a Sinusoidal Function of the Form or Using Key Points:**

**Step1:** Determine the amplitude and period of the sinusoidal function.

**Step2:** Divide the interval into four subintervals of the same length.

**Step3:** Use the endpoints of these subintervals to obtain five key points on the graph.

**Step4:** Plot the five key points, and draw a sinusoidal graph to obtain the graph of one

cycle. Extend the graph in each direction to make it complete.

**Example 6:** Draw the graph of

**Solution:** Since the sine function is odd, use the equivalent form:

**Step1:** Here,

The amplitude is = and the period is=

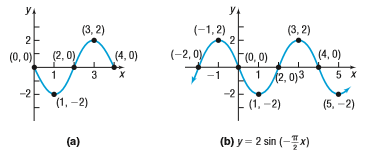
The graph lies between -2 and 2 on the *y*-axis. One cycle will begin at *x* = 0 and end at *x* = 4.

**Step2:** Divide the interval into four subintervals, each of length . The *x*-coordinates of the five key points are: 0, 1, 2, 3, 4

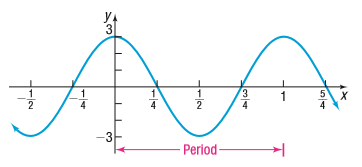
**Step3:** The five key points on the graph are:

**Step4:** Plot these five points, and fill in the graph of the sine function as shown in figure

(a) and extend the graph in each direction to obtain figure (b).



**Example 8:** Find an equation for the following graph:



**Solution:** The maximum value, 3, occurs at x = 0. So the equation can be viewed as a cosine function with *A* = 3 and period *T* = 1. Then .

The cosine function whose graph is given in the question is

Ans.

**Home Work: Exercise 6.4: Problem No. 11 - 48 and 57- 70**

**Exercise 6.4:**

**Question no. 11- 22 are same:**

**Question 14:** Determine the amplitude and period of the following function without graphing:

**Solution:** Given,

Here, Amplitude=, Period = Ans.

**Question 20:** Determine the amplitude and period of the following function without graphing:

**Solution:** Given,

Here, Amplitude=, Period = Ans.

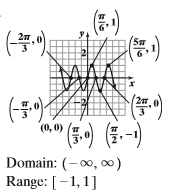
**Question no. 23- 32 are same: (Matching)**

**Question no. 33- 48 are same:**

**Question 37:** Draw the graph of the following function using transformations or the method of key points. Use the graph to determine the domain and the range:

**Solution:** Given,

Here, Amplitude = and Period =



**Question 41:** Draw the graph of the following function using transformations or the method of key points. Use the graph to determine the domain and the range:

**Solution:** Given, , Here,

